**LP Additional Practice Problems**

1. *Refrigerators.* A plant produces two types of refrigerators, A and B. There are two production lines, one dedicated to producing refrigerators of Type A, the other to producing refrigerators of Type B. The capacity of the production line for A is 60 units per day, the capacity of the production line for B is 50 units per day. A requires 20 minutes of labor whereas B requires 40 minutes of labor. Presently, there is a maximum of 40 hours of labor per day which can be assigned to either production line. Profit contributions are $20 per refrigerator of type A produced and $30 per type B produced. What should the daily production be?

GAP Words:

O: Maximize profits

D: Number of A and B to be produced

C: Total labor is limited, Production line time is limited for each type.

GAP Math:

D: A: # refrigerators of type A produced.

O: Max 20A + 30B

C: 20A + 40B <= 2400

A <= 60

B <= 50

A, B >= 0

1. *Trail Mix.* Determine how to produce bags of trail mix at minimum cost. Ingredients: **F**lakes, **P**ecans, **R**aisins, **S**eeds, **W**alnuts. The weight of each bag needs to be at least 10 lbs. The costs per pound and the nutritional requirements are summarized in the following table:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Cost / lb. | F, $3 | P, $7 | R, $5 | S, $4 | W, $6 | Requirement per package |
| Vitamins(g/lb) | 10 | 30 | 20 | 10 | 20 | at least 100 g |
| Minerals(g/lb) | 4 | 9 | 7 | 5 | 2 | at least 50 g |
| Protein(g/lb) | 10 | 2 | 4 | 1 | 1 | at least 75 g |
| Calories/lb | 160 | 300 | 450 | 500 | 500 | £ 3000 cal. |

Each package must contain at least 1 lb. of W, at least 0.75 lbs. each of P, R and S. No min quantity for flakes.

GAP Words:

O: Minimize cost

D: Flakes, Pecans, Raisins, Seeds, Walnuts per package

C: Vitamin, minerals, proteins, calories requirements, total weight requirement

GAP Math:

D: F, P, R, S, W: lbs of each ingredient per package

O: Min 3F + 7P + 5R + 4S + 6W

C: 10F + 30P + 20R + 10S + 20W >= 100

4F + 9P + 7R + 5S + 2W >= 50

10F + 2P + 4R + S + W >= 75

160F + 300P + 450R + 500S + 500W <= 3000

F + P + R + S + W >= 10

F, P, R, S, W > = 0

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Outlet** | **Boys** | **Women** | **Men** | **Cost** |
| TV | 5 | 1 | 3 | $600/ minute |
| Magazine | 2 | 6 | 3 | $500/ page |
| **Target** | **24** | **18** | **24** |  |

1. *Megamarketing.* MegaMarketing is planning a concentrated one-week advertising campaign for their new CutsEverything SuperKnife. The ads have been designed and produced and now they wish to determine how much money to spend in each advertising outlet. In reality, they have hundreds of possible outlets to choose from. We will illustrate their problem with two outlets: Prime-time TV, and newsmagazines.

The problem of optimally spending advertising dollars can be formulated in many ways. Our approach is to define targets for reaching each market segment and to minimize the money spent to reach those targets. For SuperKnife, the target segments are Teenage Boys, Affluent Women (ages 40-49), and Retired Men.

Each minute of primetime TV and page of newsmagazine advertisement reaches a certain number of people (in millions). The reach for each medium (in million people per ad minute or per ad page), the target sizes (in millions), and cost per outlet (per minute) are summarized in the table above.

How many minutes of Prime-time TV, and how many pages of news magazine should MegaMarketing purchase to meet the target segments?

GAP Words:

O: Minimize cost of advertising

D: Minutes/pages of each medium purchased

C: Target exposure for the number of boys, men and women needs to be above a certain threshold.

GAP Math:

D: T: minutes of TV ads purchased, M: pages of magazines purchased

O: Min 600T + 500M

C: 5T + 2M >= 24

T + 6M >= 18

3T + 3M >= 24

T, M > = 0

1. *Cookie Baking:* Jimmy is baking chocolate chip and oatmeal raisin cookies for sale. He gets 25 cents for each chocolate chip cookie and 30 cents for each oatmeal raisin cookie. He cannot make more than 500 cookies of each kind, and he cannot make more than 800 cookies total. He must make at least one-third as many chocolate chip cookies as oatmeal raisin cookies. How many of each kind of cookies should he make to make the most money?

GAP Words:

O: Maximize profits from cookies

D: Number of chocolate chip cookies and oatmeal raisin cookies

C: Minimum cookie numbers, maximum cookie numbers, numbers of oatmeal raisin cookies must be greater than a certain amount of chocolate chips.

GAP Math:

D: C: number of chocolate chip cookies, O: number of oatmeal raisin cookies

O: Max 25C + 30O

C: C <= 500

D <= 500

C + D <= 800

3O – C >= 0

O, C >= 0

1. *Managing a Portfolio.* A local bank wants to build a bond portfolio from a set of five bonds with $1 million available for investment. The expected annual return, the worst-case annual return on each bond, and the “duration” of each bond are given in the following table. (The duration of a bond is a measure of the bond’s sensitivity to changes in interest rates.)

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Expected Return** | **Worst Case Return** | **Duration** |
| Bond 1 | 12.5% | 8.0% | 8 |
| Bond 2 | 11.5% | 7.5% | 7 |
| Bond 3 | 10.5% | 6.8% | 6 |
| Bond 4 | 9.5% | 7.0% | 5 |
| Bond 5 | 8.5% | 7.4% | 3 |

The bank wants to maximize the expected return from its bond investments, subject to three conditions:

* The average worst-case return for the portfolio must be at least 7.2 percent.
* The average duration of the portfolio must be at most 6.
* Because of diversification requirements, at most 40 percent of the total amount invested can be invested in a single bond.

1. What is the maximum return on the $1 million investment? How should the investment be distributed among the bonds to achieve this return? (Assume that bonds can be purchased in fractional amounts.)
2. What is the marginal rate of return on the investment amount? That is, what would be the percentage return on an additional dollar invested?

GAP Words

Objective: Maximize Expected Return of Portfolio

Decision Variables: Percentage of “endowment” Invested in Each Bond

Constraints: Decision variables add up to 1, Average worst case at least 7.2 percent, Average duration at most 6, No more than 40% in a single bond.

GAP Math

Decision Variables: B1, B2, B3, B4, B5

Objective Function: Expected Return of Portfolio: B1 \* .125 + B2 \* .115 + B3 \* .105 + B4 \* .095 + B5 \* .085

Constraints: B1 + B2 + B3 + B4 + B5 = 1 (Adds up to 1)

B1 \* .08 + B2 \* .075 + B3 \* .068 + B4 \* .07 + B5 \* .074 >= .072 (Minimum Worst-Case)

B1 \* 8 + B2 \* 7 + B3 \* 6 + B4 \* 5 + B5 \* 3 <= 6 (Minimum Worst-Case)

B1, B2, B3, B4, B5 <= .4 (Maximum percent in each bond)

Answers:

1. Max Return: 10.85%, 40% in Bond 1, 25% in Bond 2, and 35% in Bond 3
2. 10.85% because all constraints are just mixing (blending) constraints. So, we would invest the extra dollar in the same proportions.
3. *Scheduling Consultants.* You are the Director of the Computer Center for Gaillard College and responsible for scheduling the staffing of the center, which is open from 8 a.m. until midnight. You have monitored the usage of the center at various times of the day and determined that the following number of computer consultants are required:

|  |  |
| --- | --- |
| **Time of Day** | **Minimum Number of Consultants** |
| 8 a.m. – noon | 4 |
| Noon – 4 p.m. | 8 |
| 4 p.m. – 8 p.m. | 10 |
| 8 p.m. - midnight | 6 |

Two types of computer consultants can be hired: full-time and part-time. The full-time consultants work for eight consecutive hours in any of the following shifts: morning (8 a.m – 4 p.m.), afternoon (noon – 8 p.m.), and evening (4 p.m. – midnight). Full-time consultants are paid $14 per hour.

Part-time consultants can be hired to work any of the four shifts listed in the table. Part-time consultants are paid $12 per hour. An additional requirement is that during every time period, there must be at least one full-time consultant on duty for every part-time consultant on duty.

Determine a minimum-cost staffing plan for the center. How many full-time consultants and part-time consultants will be needed? What is the minimum cost?

GAP Words:

Objective Function: Minimize Staffing Cost

Decision Variables: Number of Full Time Employees on their 3 Shifts, Number of part-time employees on their 4 shifts

Constraints: Minimum Coverage for each of the 4 intervals, One full-time for every part-time consultant for each of the four intervals.

GAP Math:

Decision Variables: F1, F2, F3, P1, P2, P3, P4

Objective Function: $14 per hour \* 8 hours per full-time shift \* (F1 + F2 + F3) + $12 per hour \* 4 hours per part-time shift \* (P1 + P2 + P3 + P4)

Constraints: F1 + P1 >= 4, F1 + F2 + P2 >=8, F2 + F3 + P3 >= 10, F3 + P4 >= 6 (Minimum Coverage)

F1 >= P1 🡺 F1 – P1 >= 0 (Interval 1 Full Time Mix)

F1 + F2 >= P2 🡺 F1 + F2 – P2 >=0 (Interval 2 Full Time Mix)

F2 + F3 >= P3 🡺 F2 + F3 – P3 >=0 (Interval 3 Full Time Mix)

F3 >= P4 🡺 F3 – P4 >=0 (Interval 4 Full Time Mix)

Answer: 8 Full Time, and 12 part time (see spreadsheet for breakout), Minimum cost of $1,472

1. *Coffee Blending.* As a newly appointed Chief Product Officer of Huggamug Coffee you have been tasked to develop a new coffee blend. Given current demand projections, the required minimum amount of coffee to be blended each week is 2000 tons. Further, there are three certified coffee bean suppliers that you can source from: one from Brazil (BR), one from Colombia (CO) and one from Peru (PE). Your data on past supplier performance is that the aroma scores of the three suppliers are 75, 60 and 85, respectively. Further, the strength scores of the three suppliers are 15, 20 and 18, respectively. Lastly, each week the three suppliers can deliver 750, 600 and 1,000 tons, at the cost of 1000, 1200 and 1400 $/ton, respectively. Your blend must have an aroma score of at least 78 and a strength score of at least 16.75. What blend would achieve these performance criteria at a minimum cost?

GAP WORDS

Objective Function: Minimize cost of blend

Decision Variables: Tons of coffee to be procured from each supplier (Be, Co, Pe)

Constraints: Minimum Tons Produced, Maximum Available of Each Type, Minimum Aroma Rating, Minimum Strength

GAP MATH

Decision Variables: Brazilian (Columbian, Peruvian) Tons = Br, Co, Pe

Objective Function:

Coffee Production Cost = Brazilian Tons \* Brazilian Cost/Ton + Columbian Tons \* Columbian Cost/Ton + Peruvian Tons \* Peruvian Cost/Ton

Coffee Production Cost = Br \* 1,000 + Co \* 1,200 + Pe \* 1,400

Constraints (6 total):   
Br + Co + Pe >= 2000  
Br <= 750  
Co <= 600  
Pe <= 1000

, equivalent to

, equivalent to

, equivalent to

, equivalent to

1. *Transportation:* The following table shows the transportation costs (in $/item) from four factories to three stores, as well as the demand at each store, and the maximum supply of each factory. Determine the minimum cost shipping plan that satisfied the demand and supply requirements.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **Per unit transportation costs** | | | | | |
|  |  | Stores | | | |  |
|  |  | Atl | Bos | Chi | Den | Supply |
| Factories | Minn | 0.60 | 0.56 | 0.22 | 0.40 | 9000 |
| Pitt | 0.36 | 0.30 | 0.28 | 0.58 | 12000 |
| Tucs | 0.65 | 0.68 | 0.55 | 0.42 | 13000 |
| Demand | | 7500 | 8500 | 9500 | 8000 |  |

GAP Words:

O: minimize transportation costs

D: numbers of items ship from factories to stores

C: supply limit; demand limit

GAP Math:

D: MA (Minn to Atl), PA, TA, MB, PB, TB, MC, PC, TC, MD, PD, TD

O: Minimize 0.60 \* MA + 0.36 \* PA + 0.65 \* TA + 0.56 \* MB + 0.30 \* PB + 0.68 \* TB + 0.22 \* MC + 0.28 \* PC + 0.55 \* TC + 0.40 \* MD + 0.58 \* PD + 0.42 \* TD

C: MA + MB + MC + MD <= 9000

PA + PB + C + PD <= 12000

TA + TB + TC + TD <= 13000

MA + PA + TA >= 7500

MB + PB + TB >= 8500

MC + PC + TC >= 9500

MD + PD + TD >= 8000

Non-negative constraints for all DVs

*9) Burgers* The Green New Deal Burger Co is exploring new sources of meatless meat. To attract Keto/Paleo consumers, the company is using large amounts of soy protein and a small amount of carbs in their burger patties. To achieve profitability, the key decision in their manufacturing process is thus where to source processed soy beans. Currently, the available soy suppliers are willing to supply soy in the following amounts and at the following price:

Friendly Farms (FF): at most 200 lbs at $4/lbs

Misty Meadows (MM): at most 310 lbs at $3/lbs

Treasure Tree (TT): at most 420 lbs at $2/lbs

Shipping costs in $ per lbs are:

To: Plant DC-A Plant DC-B

From: FF 3 3.5

MM 2 2.5

TT 6 4

Fridge capacity for meat storage and labor costs at the production plants are as follows:

Plant DC-A Plant DC-B

Capacity 460 lbs 560 lbs

Labor cost $26/lb $21/lb

Each patty is sold at $5 and requires 0.15 lbs of processed soy. The company can sell at this price all they can produce.

1. What is the best mixture of the quantities supplied by the three suppliers to the two plants so that the company maximizes its profits? Is the company profitable?

**GAP:**

**Nominal language:**

Objective: Maximize profit from selling burger patties.

Decision variables: Amount to purchase and transport from each supplier to each plant

Constraints: Supply limits for suppliers; Plant A&B capacities.

**Math:**

Decision variables: FFA, MMA, TTA, FFB, MMB, TTB.

For example, FFA stands for lbs of soy from supplier FF to plant DC-A, etc..

Objective function:

Max Revenue – Cost,

where we can plug in the following expressions for Revenue and cost:

Revenue = 5 \* (FFA+MMA+TTA+FFB+MMB+TTB)/0.15

Cost = 26 \* (FFA+MMA+TTA) – 21 \* (FFB+MMB+TTB) – (4 + 3) \* FFA – (3 + 2) \* MMA –   
(2 + 6) \* TTA – (4 + 3.5) \* FFB – (3 + 2.5) \* MMB – (2 + 4) \* TTB

*Note that Costs include (i) labor cost at each plant, (ii) sourcing costs from each supplier, and (iii) transportation costs from suppliers to plants.*

Constraints:

FFA + MMA + TTA <= 460

FFB + MMB + TTB <= 560

FFA + FFB <= 200

MMA + MMB <= 310

TTA + TTB <= 420

FFA, MMA, TTA, FFB, MMB, TTB >= 0

Answer:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Variable | FFA | MMA | TTA | FFB | MMB | TTB |
| Ibs | 200 | 170 | 0 | 0 | 140 | 420 |

Maximum profit is $4080 (See Excel solution)

1. The combative Keto/Paleo-community is demanding that the carb content in the patty be further reduced to less than 1%, which would require the amount of soy per patty to go up to 0.2 lbs. Assuming that prices are not allowed to increase, is the venture still profitable?

We must change the objective function as follows (other components of GAP remain unchanged):

5 \* (FFA+MMA+TTA+FFB+MMB+TTB)/**0.2** – 26 \* (FFA+MMA+TTA) – 21 \* (FFB+MMB+TTB+200PPB) – (4 + 3) \* FFA – (3 + 2) \* MMA – (2 + 6) \* TTA – (4 + 3.5) \* FFB – (3 + 2.5) \* MMB – (2 + 4) \* TTB

With this objective function, the optimal solution is to transport 0 tons of soy for each possible combination of warehouse and plant, which means that we’d rather stop operations than continue sourcing from these suppliers. Hence, the venture is no longer profitable.

1. You were able to identify an additional soybean supplier, Pretty Pastures (PP), who is willing to supply *exactly* 200 lbs of soy for $1000 (PP will not supply any other quantity than 200lbs). If their offer is accepted, PP will ship directly to plant B at no additional cost. Do you accept their offer?

We need to define the additional binary variable:

PPB = 0 if we don’t use PP supplier or PPB = 1 if we use PP supplier.

Then, Objective function becomes:

5 \* (FFA+MMA+TTA+FFB+MMB+TTB+200\*PPB)/**0.2** – 26 \* (FFA+MMA+TTA) – 21 \* (FFB+MMB+TTB) – (4 + 3) \* FFA – (3 + 2) \* MMA – (2 + 6) \* TTA – (4 + 3.5) \* FFB – (3 + 2.5) \* MMB – (2 + 4) \* TTB – 1000PPB

1. *Transshipment: Your company owns* three factories: F1, F2, F3. and five warehouses: W1, W2, W3, W4, W5. Goods are shipped from factories to two depots (D1, D2), repackaged and then shipped to the warehouses. Shipping costs, capacities and demands are shown below. Find the shipping plan with minimum cost.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| From factories | D1 | D2 | W6 | Capacity |
| F1 | 1.28 | 1.36 | 1.73 | 2500 |
| F2 | 1.33 | 1.38 | 1.91 | 3000 |
| F3 | 1.68 | 1.55 | 1.86 | 2300 |

Warehouse 6 (W6) demand = 800

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| From depots | W1 | W2 | W3 | W4 | W5 |
| D1 | 0.60 | 0.42 | 0.32 | 0.44 | 0.68 |
| D2 | 0.57 | 0.30 | 0.40 | 0.38 | 0.72 |
| Demand | 1200 | 1300 | 1400 | 1500 | 1600 |

GAP words:

O: minimize transportation costs

D: Amounts of goods shipped from factories to distribution centers and W6, and from distribution centers to warehouses

C: factories' capacities, warehouse demand, inbound logistics should be equal to outbound logistic for distribution centers

GAP math:

D: F1D1 (from F1 to D1), F2D1, F3D1, F1W6 (from F1 to W6), F2W6, F3W6, F1D2, F2D2, F3D2, D1W1, D2W1, D1W2, D2W2, D1W3, D2W3, D1W4, D2W4, D1W5, D2W5

O: min 1.28 \* F1D1 + 1.33 \* F2D1 + 1.68 \* F3D1 + 1.73 \* F1W6 + 1.91 \* F2W6 + 1.86 \* F3W6 + 1.36 \* F1D2 + 1.38 \* F2D2 + 1.55 \* F3D2 + 0.60 \* D1W1 + 0.57 \* D2W1 + 0.42 \* D1W2 + 0.30 \* D2W2 + 0.32 \* D1W3 + 0.40 \* D2W3 + 0.44 \* D1W4 + 0.38 \* D2W4 + 0.68 \* D1W5 + 0.72 \* D2W5

C: F1D1 + F1D2 + F1W6 <= 2500

F2D1 + F2D2 + F2W6 <= 3000

F3D1 + F3D2 + F3W6 <= 2300

D1W1 + D2W1 >= 1200

D1W2 + D2W2 >= 1300

D1W3 + D2W3 >= 1400

D1W4 + D2W4 >= 1500

D1W5 + D2W5 >= 1600

F1W6 + F2W6 + F3W6 >= 800

(F1D1 + F2D1 + F3D1) - (D1W1 + D1W2 + D1W3 + D1W4 + D1W5) = 0

(F1D2 + F2D2 + F3D2) - (D2W1 + D2W2 + D2W3 + D2W4 + D2W5) = 0